AN831

Matching Small Loop Antennas to rfPIC™ Devices

Author: Jan van Niekerk
Microchip Technology Inc.

INTRODUCTION

In close proximity to the human body, small loop antennas outperform small dipole and monopole antennas [1]. Their size, robustness and low manufacturing cost have made small loops the most popular antenna for use in miniature key fob transmitters. A small loop antenna typically consists of a circular, square or rectangular copper trace on a printed circuit board. In some cases, discrete wires are shaped into loops.

FIGURE 1: EQUIVALENT CIRCUIT MODEL OF A SMALL LOOP ANTENNA

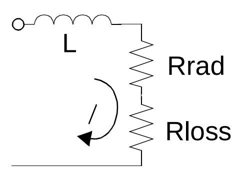


Figure 1 shows an equivalent circuit of a loop antenna consisting of two resistors and an inductor. The resistor *Rrad*, or radiation resistance, models the radio frequency energy actually radiated by the antenna. *Rrad* models the desired function of the antenna, which is to radiate RF power. Assuming a uniform current I flowing through the loop, the power consumed by *Rrad* (i.e., the radiated power) is shown in Equation 1.

EQUATION 1:

$$Pradiated = I^2 \cdot Rrad$$

The second resistor in the model, *Rloss*, models losses. *Rloss* models an undesired, but inevitable function of the antenna: to waste valuable RF energy by converting it to heat. If *Rloss* is larger than *Rrad*, the antenna is inefficient, since most of the available RF power will end up as heat. With current I flowing through the loop, the lost power (converted to heat) is given by Equation 2.

EQUATION 2:

$$Ploss = I^2 \cdot Rloss$$

Note that we assume that the current I is uniform around the small loop. This assumption is only valid if the loop circumference is smaller than one fifth of a wavelength.

For completeness, note that the total power delivered to the antenna is given by the sum of the radiated power and losses. From Equation 1 and Equation 2, we get Equation 3:

EQUATION 3:

$$Ptotal = Pradiated + Ploss = I^2 \cdot (Rrad + Rloss)$$

In practice, the loop antenna designer has little control over *Rrad* and *Rloss*. *Rrad* is determined by the area of the loop antenna and *Rloss* is a function of conductor size and conductivity, as shown in Equation 4 and 5.

CALCULATING THE LOOP RADIATION RESISTANCE AND LOSS RESISTANCE

The radiation resistance *Rrad* of a small loop antenna is given by reference [2] as:

EQUATION 4:

$$Rrad = 31171 \left(\frac{A^2}{\lambda^4}\right)$$

where A is the area of the loop in square meter and λ is the wavelength in meters at the radiation frequency. It should be clear from Equation 4 that the radiation resistance of small loops will be in the milliohm range. The wavelength λ can be calculated as $\lambda = 3 \cdot 10^8 / f$ where f is the radiating frequency in Hertz.

The loss resistance *Rloss* of a loop antenna is given by reference [2] as:

EQUATION 5:

$$Rloss = \frac{l}{2w} \cdot \sqrt{\frac{\pi f \mu}{\sigma}}$$

where l is the perimeter (circumference) of the loop in meters, w is the width of the PCB track in meters, f is the radiating frequency in Hertz, $\mu = 4\pi\cdot10^{-7}$ and σ is the conductivity of the PCB track in Siemens per meter. Copper conductivity is typically $5.7\cdot10^{7}$ S/m.

Equation 5 is essentially the result of the 'skin effect' [2] at high frequency for nonmagnetic materials. In this case, the perimeter of the conductor, normally $2\pi r$ for a round wire, has been approximated by 2w. In other words, its perimeter is 2 times the PCB trace width.

CALCULATING THE INDUCTANCE OF THE LOOP

The third component in the model of Figure 1 is the loop inductance L. Inductance is primarily a magnetic effect, and general inductance formulas for even simple shapes are hard to derive. Several formulas for calculating the inductance of rectangular loops have been proposed. Most of these formulas are lengthy [2,3,4]. Grover's book [3], which is the primary reference work on inductance, provides one remarkably simple, but accurate formula for calculating the inductance of polygons. This formula includes, but is not limited to rectangular loops. The inductance formula given by Grover [3] is:

EQUATION 6:

$$L = \frac{\mu}{2\pi}\cdot l\cdot \ln\left(\frac{8\cdot A}{l\cdot w}\right)$$

where $\mu = 4\pi\cdot10^{-7}$, A is the area of the loop in square meters, l is the perimeter (circumference) of the loop in meters, and w is the width of the copper trace in meters.

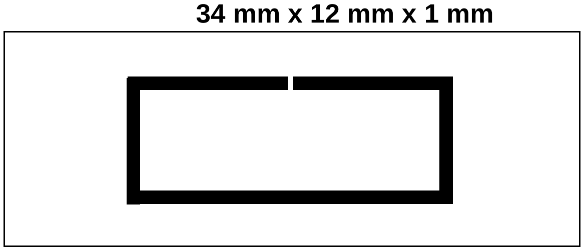
CALCULATION OF LOOP PARAMETERS

EXAMPLE 1:

Suppose a designer is constrained by PCB loop antenna dimensions of 34 mm x 12 mm. The copper track width is 1 mm.

The total loop resistance, that is the sum of radiation resistance and loss resistance, is calculated to be 0.249 Ω.

FIGURE 2: PCB COPPER LOOP 34 mm x 12 mm x 1 mm



Using Equation 4, we calculate radiation resistance at 434 MHz as *Rrad* = 0.0227 Ω.

Using Equation 5, we calculate loss resistance at 434 MHz as *Rloss* = 0.252 Ω (σ of copper $5.7*10^{7}$).

Using Equation 6, we calculate loop inductance as L = 65.67 nH.

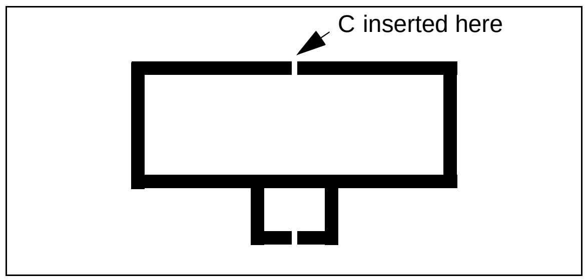
Summing the loss resistance and radiation resistance, total loop resistance is calculated to be r = 0.275 Ω.

Matching the Loop to a 1 kΩ Source Impedance

A typical CMOS radio frequency integrated circuit, such as the rfPIC12C509AG, has a source impedance around 1 kΩ. In the example above, the impedance of a typical loop has an inductance of 65.67 nH in series with a small resistance of 0.275 Ω. To match such an antenna to the source, this low resistance and high inductive reactance must be transformed to 1 kΩ.

The impedance transformation required is achieved by adding a second, smaller loop to our antenna, as well as a capacitor C, as shown in Figure 3.

FIGURE 3: ADDING A SMALL SECOND LOOP AND CAPACITOR



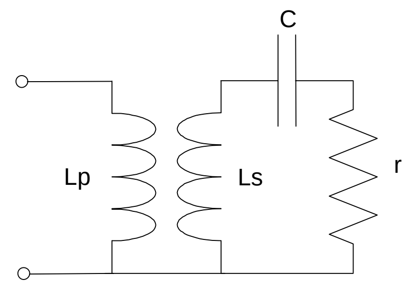
The magnetic coupling between the large loop and small loop results in transformer action. The large loop, or loop antenna, makes up the secondary winding of our transformer. The small loop becomes the primary winding of our transformer.

Figure 4 is a revised circuit model of the loop antenna, showing the transformer action. The loop antenna's total resistance r, consisting of *Rrad* + *Rloss*, forms the resistive load in the secondary circuit. Also note a capacitance *C* in the secondary circuit. Capacitor C is primarily used to cancel the loop inductance L_S. The capacitance may be approximated as follows:

EQUATION 7:

$$C \approx \frac{1}{L_S\omega^2}$$

FIGURE 4: REVISED LOOP ANTENNA EQUIVALENT CIRCUIT



MAGNETIC COUPLING

Magnetic coupling between the primary and secondary windings is at the root of the impedance transformation. We now write the basic voltage and current equations for the above magnetically coupled circuit:

EQUATION 8:

$$V_p = j\omega L_p I_p + j\omega M I_s$$

$$0 = j\omega M I_p + j\omega L_s I_s - j\frac{1}{\omega C} I_s + r I_s$$

where V_p is the primary voltage, I_p the primary current, I_s the secondary current and ω the angular frequency, equal to $2 \cdot \pi \cdot f$. M is the mutual inductance, which is a function of the degree of magnetic coupling between the two loops.

By using Equation 8, the real part (resistive portion) of the load impedance as seen from the primary side of the transformer can be derived:

EQUATION 9:

$$Re(Z_p) = (\omega M)^2 \cdot \frac{r}{\left[r^2 + \left(\omega L_s - \frac{1}{\omega C}\right)^2\right]}$$

Near resonance the term

$$\left(\omega L_s - \frac{1}{\omega C}\right)^2$$

becomes small, so that it is possible to estimate the resistive load as seen from the primary side by:

EQUATION 10:

$$R_p \approx (\omega M)^2 \cdot \frac{1}{r}$$

Equation 10 shows how the transformer action translates the low loop resistance r of the small loop antenna:

- The resistance is inverted.
- The inverted resistance is then multiplied by the square of the mutual reactance, $(\omega M)^2$.

Equation 10 can be rewritten as:

EQUATION 11:

$$M \approx \frac{\sqrt{r \cdot Rp}}{\omega}$$

Equation 11 shows the mutual inductance needed to transform a loop impedance of r ohm into a needed source impedance of R_p ohm.

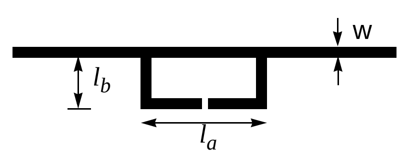
Next, we find a way to calculate mutual inductance M as a function of loop dimensions.

Obtaining a Given Mutual Inductance

A formula for the calculation of mutual inductance between two off-center, coplanar rectangles is daunting. However, two reasonable assumptions simplify the calculation significantly. The assumptions are:

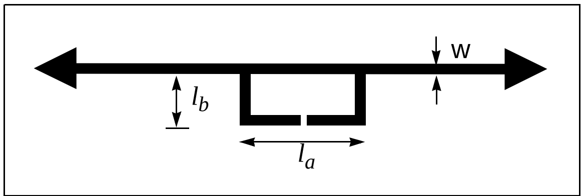
1. Assume that only one side of the loop antenna couples magnetically to the small loop. The other three sides are much further away, so we may neglect their effect. This assumption simplifies the mutual inductance calculation problem to that of mutual inductance between a straight wire (or PCB track), and the small loop, as drawn in Figure 5.

FIGURE 5: USING ONLY ONE SIDE OF THE LARGE LOOP



2. It is assumed that the straight wire of Figure 5 stretches to infinity, as drawn in Figure 6. This is reasonable because the part of the wire close to the loop is the dominating contributor of magnetic flux in the small loop.

FIGURE 6: INFINITE WIRE AND SMALL LOOP



The two assumptions greatly simplify the calculation of mutual inductance. The mutual inductance of the loop-and-wire of Figure 6 is a popular college physics [5] problem with a compact result:

EQUATION 12:

$$M = \frac{\mu}{2\pi} \cdot l_a \cdot \ln\left(1 + \frac{4 \cdot l_b}{w}\right)$$

Where M is mutual inductance in Henry, l_a is the rectangle dimension parallel to the wire, l_b is the rectangle dimension perpendicular to the wire, and w is the width of the PCB track. All dimensions are in meter, and $\mu = 4\pi \cdot 10^{-7}$.

By setting the rectangle dimension l_b to 2 times the PCB track width, in other words, setting l_b = 2w, we find that Equation 12 simplifies to:

EQUATION 13:

$$M = \frac{\mu}{2\pi} \cdot l_a \cdot \ln\left(1 + \frac{8w}{w}\right)$$

$$M = \frac{\mu}{2\pi} \cdot l_a \cdot \ln(1 + 8)$$

By combining Equations 11 and 13, an expression for loop dimension l_a is found as follows:

EQUATION 14:

$$l_a = \frac{\sqrt{r \cdot Rp}}{\mu \cdot f \cdot \ln(9)}$$

Equation 14 is the final result and provides a simple method to match to a small loop antenna, which is summarized below.

1. Calculate the loop series resistance r using Equations 4 and 5.
2. Determine the required impedance R_p.
3. Calculate l_a of a small matching loop using Equation 14.

EXAMPLE 2:

Continuing our loop antenna of Example 1:

1. From Example 1, we calculated the loop series resistance as r = 0.275 Ω
2. The needed antenna impedance is 1 kΩ.
3. From Equation 14, we calculate l_a = 13.8 mm, using f = 434 MHz.

CONCLUSION

A simple method to match a small loop antenna has been found. By adding a small primary loop and by controlling the mutual inductance of the resulting transformer, the low loop resistance is transformed to the value desired for maximum power transfer.

REFERENCES

[1] K. Fujimoto and J.R. James, *Mobile Antenna Systems,* Second Edition Artech House 2001 ISBN 1-58053-007-9

[2] K. Fujimoto, A. Henderson, K. Hirasawa and J.R. James, *Small Antennas,* Research Studies, Press LTD 1987 ISBN 0 86380 048 3 (Research Studies Press) ISBN 0 471 91413 4 (John Wiley & Sons Inc)

[3] Frederick W. Grover, *Inductance Calculations Working Formulas and Tables,* Dover Publications Inc. 1946

[4] V.G. Welsby, *The Theory and Design of Inductance Coils,* 2nd Edition John Wiley & Sons Inc. 1960

[5] Matthew N.O. Sadiku, *Elements of Electromagnetics,* Third Edition Oxford University Press 2001 ISBN 0-19-513477-X

[6] Thomas H. Lee, *The Design of CMOS Radio-Frequency Integrated Circuits,* Cambridge University Press 1998 ISBN 0-521-63061-4 (hb) ISBN 0-521-63922-0 (pb)

[7] Chris Bowick, *RF Circuit Design,* H.W. Sams 1982 ISBN 0-7506-9946-9

APPENDIX A: COMPLEX IMPEDANCE

Equation 9 shows only the real part of the primary side impedance. The entire complex impedance as seen on the primary side is:

EQUATION 15:

$$Z_p = \omega^2 M^2 \cdot \frac{r}{\left[r^2 + \left(\omega L_s - \frac{1}{\omega C}\right)^2\right]} + j\left[\omega L_p - \frac{\omega^2 M^2 \cdot \left(\omega L_s - \frac{1}{\omega C}\right)}{\left[r^2 + \left(\omega L_s - \frac{1}{\omega C}\right)^2\right]}\right]$$

M is mutual inductance, r is loop resistance, L_s is secondary (large) loop inductance and L_p is primary loop inductance. L_p and L_s are both calculated using Equation 6.

An exact value for the capacitance C at resonance can be found by setting the imaginary part of Equation 15 to zero.

Note the following details of the code protection feature on PICmicro® MCUs.

- The PICmicro family meets the specifications contained in the Microchip Data Sheet.
- Microchip believes that its family of PICmicro microcontrollers is one of the most secure products of its kind on the market today, when used in the intended manner and under normal conditions.
- There are dishonest and possibly illegal methods used to breach the code protection feature. All of these methods, to our knowledge, require using the PICmicro microcontroller in a manner outside the operating specifications contained in the data sheet. The person doing so may be engaged in theft of intellectual property.
- Microchip is willing to work with the customer who is concerned about the integrity of their code.
- Neither Microchip nor any other semiconductor manufacturer can guarantee the security of their code. Code protection does not mean that we are guaranteeing the product as “unbreakable”.
- Code protection is constantly evolving. We at Microchip are committed to continuously improving the code protection features of our product.

If you have any further questions about this matter, please contact the local sales office nearest to you.

Printed on recycled paper.

Microchip received QS-9000 quality system certification for its worldwide headquarters, design and wafer fabrication facilities in Chandler and Tempe, Arizona in July 1999. The Company’s quality system processes and procedures are QS-9000 compliant for its PICmicro® 8-bit MCUs, KEELOQ® code hopping devices, Serial EEPROMs and microperipheral products. In addition, Microchip’s quality system for the design and manufacture of development systems is ISO 9001 certified.

MICROCHIP

WORLDWIDE SALES AND SERVICE

AMERICAS

Corporate Office
2355 West Chandler Blvd.
Chandler, AZ 85224-6199
Tel: 480-792-7200 Fax: 480-792-7277
Technical Support: 480-792-7627
Web Address: http://www.microchip.com

Rocky Mountain
2355 West Chandler Blvd.
Chandler, AZ 85224-6199
Tel: 480-792-7966 Fax: 480-792-7456

Atlanta
500 Sugar Mill Road, Suite 200B
Atlanta, GA 30350
Tel: 770-640-0034 Fax: 770-640-0307

Boston
2 Lan Drive, Suite 120
Westford, MA 01886
Tel: 978-692-3848 Fax: 978-692-3821

Chicago
333 Pierce Road, Suite 180
Itasca, IL 60143
Tel: 630-285-0071 Fax: 630-285-0075

Dallas
4570 Westgrove Drive, Suite 160
Addison, TX 75001
Tel: 972-818-7423 Fax: 972-818-2924

Detroit
Tri-Atria Office Building
32255 Northwestern Highway, Suite 190
Farmington Hills, MI 48334
Tel: 248-538-2250 Fax: 248-538-2260

Kokomo
2767 S. Albright Road
Kokomo, Indiana 46902
Tel: 765-864-8360 Fax: 765-864-8387

Los Angeles
18201 Von Karman, Suite 1090
Irvine, CA 92612
Tel: 949-263-1888 Fax: 949-263-1338

New York
150 Motor Parkway, Suite 202
Hauppauge, NY 11788
Tel: 631-273-5305 Fax: 631-273-5335

San Jose
Microchip Technology Inc.
2107 North First Street, Suite 590
San Jose, CA 95131
Tel: 408-436-7950 Fax: 408-436-7955

Toronto
6285 Northam Drive, Suite 108
Mississauga, Ontario L4V 1X5, Canada
Tel: 905-673-0699 Fax: 905-673-6509

ASIA/PACIFIC

Australia
Microchip Technology Australia Pty Ltd
Suite 22, 41 Rawson Street
Epping 2121, NSW
Australia
Tel: 61-2-9868-6733 Fax: 61-2-9868-6755

China - Beijing
Microchip Technology Consulting (Shanghai) Co., Ltd., Beijing Liaison Office
Unit 915
Bei Hai Wan Tai Bldg.
No. 6 Chaoyangmen Beidajie
Beijing, 100027, No. China
Tel: 86-10-85282100 Fax: 86-10-85282104

China - Chengdu
Microchip Technology Consulting (Shanghai) Co., Ltd., Chengdu Liaison Office
Rm. 2401, 24th Floor,
Ming Xing Financial Tower
No. 88 TIDU Street
Chengdu 610016, China
Tel: 86-28-6766200 Fax: 86-28-6766599

China - Fuzhou
Microchip Technology Consulting (Shanghai) Co., Ltd., Fuzhou Liaison Office
Unit 28F, World Trade Plaza
No. 71 Wusi Road
Fuzhou 350001, China
Tel: 86-591-7503506 Fax: 86-591-7503521

China - Shanghai
Microchip Technology Consulting (Shanghai) Co., Ltd.
Room 701, Bldg. B
Far East International Plaza
No. 317 Xian Xia Road
Shanghai, 200051
Tel: 86-21-6275-5700 Fax: 86-21-6275-5060

China - Shenzhen
Microchip Technology Consulting (Shanghai) Co., Ltd., Shenzhen Liaison Office
Rm. 1315, 13/F, Shenzhen Kerry Centre,
Renminnan Lu
Shenzhen 518001, China
Tel: 86-755-2350361 Fax: 86-755-2366086

Hong Kong
Microchip Technology Hongkong Ltd.
Unit 901-6, Tower 2, Metroplaza
223 Hing Fong Road
Kwai Fong, N.T., Hong Kong
Tel: 852-2401-1200 Fax: 852-2401-3431

India
Microchip Technology Inc.
India Liaison Office
Divyasree Chambers
1 Floor, Wing A (A3/A4)
No. 11, O'Shaugnessey Road
Bangalore, 560 025, India
Tel: 91-80-2290061 Fax: 91-80-2290062

Japan
Microchip Technology Japan K.K.
Benex S-1 6F
3-18-20, Shinyokohama
Kohoku-Ku, Yokohama-shi
Kanagawa, 222-0033, Japan
Tel: 81-45-471- 6166 Fax: 81-45-471-6122

Korea
Microchip Technology Korea
168-1, Youngbo Bldg. 3 Floor
Samsung-Dong, Kangnam-Ku
Seoul, Korea 135-882
Tel: 82-2-554-7200 Fax: 82-2-558-5934

Singapore
Microchip Technology Singapore Pte Ltd.
200 Middle Road
#07-02 Prime Centre
Singapore, 188980
Tel: 65-6334-8870 Fax: 65-6334-8850

Taiwan
Microchip Technology Taiwan
11F-3, No. 207
Tung Hua North Road
Taipei, 105, Taiwan
Tel: 886-2-2717-7175 Fax: 886-2-2545-0139

EUROPE

Denmark
Microchip Technology Nordic ApS
Regus Business Centre
Lautrup hoj 1-3
Ballerup DK-2750 Denmark
Tel: 45 4420 9895 Fax: 45 4420 9910

France
Microchip Technology SARL
Parc d'Activite du Moulin de Massy
43 Rue du Saule Trapu
Batiment A - ler Etage
91300 Massy, France
Tel: 33-1-69-53-63-20 Fax: 33-1-69-30-90-79

Germany
Microchip Technology GmbH
Gustav-Heinemann Ring 125
D-81739 Munich, Germany
Tel: 49-89-627-144 0 Fax: 49-89-627-144-44

Italy
Microchip Technology SRL
Centro Direzionale Colleoni
Palazzo Taurus 1 V. Le Colleoni 1
20041 Agrate Brianza
Milan, Italy
Tel: 39-039-65791-1 Fax: 39-039-6899883

United Kingdom
Arizona Microchip Technology Ltd.
505 Eskdale Road
Winnersh Triangle
Wokingham
Berkshire, England RG41 5TU
Tel: 44 118 921 5869 Fax: 44-118 921-5820

03/01/02